Decentralized delayed-feedback control of a coupled map model for open flow

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(Received 11 February 1998; revised manuscript received 2 June 1998)

A decentralized delayed-feedback control of a coupled map model for open flow is studied theoretically. The decentralized controllers can suppress turbulent behavior in the open flow model. The stability of the control system is analyzed, and then we provide a procedure to design the decentralized controllers from uncertain information of the local map and the coupling strength. The theoretical results are verified by some numerical simulations. [S1063-651X(98)06209-6]

PACS number(s): 05.45.+b

I. INTRODUCTION

Ott, Grebogi, and Yorke proposed a method (the OGY method) that stabilizes chaotic motions onto a desired unstable periodic orbit (UPO) [1]. The OGY method has generated great interest in the study of nonlinear science [2]. On the other hand, Pyragas proposed a delayed-feedback control (DFC) method that does not require a reference signal corresponding to the desired UPO [3]. The DFC method is a practical tool for stabilizing real chaotic systems. As a consequence, this method has been successfully applied to several physical systems: laser systems [4,5], electronic circuits [6], thin yttrium iron garnet films [7] and a magnetomechanical system [8]. The stability of the DFC systems has been analyzed by several researchers [9–11]. Furthermore, a discrete-time version of this method has been studied in detail [4,12–15]. Most of these studies on the OGY and DFC methods dealt with temporal chaotic behavior in low-dimensional systems.

In recent years, investigations of spatiotemporal chaotic behavior and studies on controlling spatiotemporal chaotic systems have attracted much interest [16–30]. Spatiotemporal chaos occurs in partial different equations (PDE's), coupled ordinary equations (COE's) or coupled map lattices (CML's). Although PDE's and COE's are exact models of real systems, the theoretical and numerical analyses of them is difficult. Conversely, it is easy to analyze the behavior of the CML's in theoretical and numerical works, since it has discrete time, discrete space, and continuous state variables. The CML's cannot identify real systems exactly; however, it can display a behavior qualitatively similar to that of more realistic models. Hence the CML's have been widely investigated by many researchers [22]. In particular, various techniques have been proposed to control the CML's [23–30]. On the basis of the OGY approach, Astakhov, Anishchenko, and Shabunin proposed a method for controlling a chain of the logistic maps [23]. Auerbach showed how an unsymmetrical CML can be controlled to behave periodically by the distributed controllers at several spatial locations [24]. Konishi and Kokame showed that their learning control technique can stabilize the CML's [25]. A feedback pinning technique for controlling the CML's has been discussed theoretically [26,27] and numerically [28]. Parmananda and Jiang proposed a control technique that uses the difference between a local state and a global average of the CML's [29]. Parmananda, Hildebrand, and Eiswirth proposed several techniques based on the OGY and DFC methods [30].

The bifurcations and pattern dynamics in an open flow model described by a one-way coupled map lattice (OCML) have been studied in detail [31,32]. The synchronization of the OCML has been investigated by the positive conditional Lyapunov exponents [33]. The synchronization has been applied to multichannel spread-spectrum communication [34]. The suppression of turbulent behavior in the OCML corresponds to transformation of open flow systems from a turbulent state to a laminar flow; hence, from a practical viewpoint, the stabilization of the homogeneous state in the OCML is an important issue. The paucity of reports on this issue prompted us to investigate it.

The purpose of the present paper is to propose a decentralized delayed-feedback control (DDFC) to stabilize an unstable homogenous state of the OCML. This control system does not require information about the unstable homogenous state. For every local site a delayed-feedback controller is used as a local controller. Each of the local controllers does not require information about other sites; therefore, it would be easy to apply our control to practical open flow systems. This paper analyzes the stability of the control system theoretically, and gives a solution for the problem of designing the local controllers. Very recently, Ishi, Konishi, and Kokame discussed a robust control problem for the extended delayed-feedback control system [14]. We also discuss the robust control problem for the OCML, and show some numerical examples to verify the theoretical results.

This paper is organized as follows. Section II describes the OCML and the control system. In Sec. III, we discuss the stability of the control system and give a procedure to design the robust local controllers. In Sec. IV, we use the logistic map as the local map and present some numerical simulations. Finally, conclusions are presented in Sec. V.

II. CONTROL SYSTEM

Let us consider a one-way coupled map lattice with size N (N-OCML),

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$$x_{n+1}(1)=f[x_n(1)],$$
$$x_{n+1}(i)=(1-\varepsilon)f[x_n(i)]+\varepsilon f[x_n(i-1)] \quad (i=2,3,\ldots,N), \tag{1}$$

where n is the discrete time, i is the lattice site, $x_n(i)$ is the system state, N is the system size, f is an unknown one-dimensional local map, and the parameter $\varepsilon \in (0,1)$ is the coupling strength. In the case of $\varepsilon=0$, each local site behaves independently. The N-OCML has an unstable homogeneous state

$$[x_n(1) \;\; x_n(2) \;\; \cdots \;\; x_n(N)]^T=[x_f \;\; x_f \;\; \cdots \;\; x_f]^T, \tag{2}$$

$$x_f=f(x_f), \tag{3}$$

where x_f is the unstable fixed point (UFP) of the local map f. Note that each element of Eq. (2) is the UFP of the local map. The purpose of this paper is that all sites converge on x_f, that is

$$\lim_{n\to+\infty} x_n(i)=x_f \quad (i=1,2,\ldots,N),$$

by the local delayed-feedback controllers. We consider a DDFC system

$$\begin{aligned} x_{n+1}(1)&=f[x_n(1)]+u_n(1),\\ x_{n+1}(i)&=(1-\varepsilon)f[x_n(i)]+\varepsilon f[x_n(i-1)]\\ &\quad +u_n(i) \quad (i=2,3,\ldots,N). \end{aligned} \tag{4}$$

The local controllers are given by

$$\begin{aligned} u_n(1)&=k_l[x_n(1)-x_{n-1}(1)],\\ u_n(i)&=k[x_n(i)-x_{n-1}(i)] \quad (i=2,3,\ldots,N), \end{aligned} \tag{5}$$

where k_l, k are the feedback gains. Notice that each local controller does not use other site information. In Sec. III, we shall analyze the stability of a DDFC system consisting of Eqs. (4) and (5).

III. STABILITY ANALYSIS

In the OCML, it should be noted that the local site $x_n(i)$ is influenced only by the *previous* site $x_n(i-1)$. Now let us assume that all the sites $x_n(i)$ $(i=2,3,\ldots,N)$ will be stabilized in the order of the site number: a local site $x_n(i)$ will be stabilized onto x_f after the previous site $x_n(i-1)$ is stabilized. The above assumption simplifies the stability analysis.

To begin with, we shall focus on the stability of the first site $x_n(1)$. Since the first site is not influenced by other sites, it is easy to derive the stability condition. The first site system can be linearized as

$$\begin{bmatrix} y_{n+1}(1)\\ z_{n+1}(1)\end{bmatrix}=\begin{bmatrix} \Lambda+k_l & -k_l\\ 1 & 0\end{bmatrix}\begin{bmatrix} y_n(1)\\ z_n(1)\end{bmatrix}, \tag{6}$$

$$y_n(1)=x_n(1)-x_f, \quad z_n(1)=x_{n-1}(1)-x_f, \tag{7}$$

where

$$\Lambda=\left.\frac{\partial f(x)}{\partial x}\right|_{x=x_f} \tag{8}$$

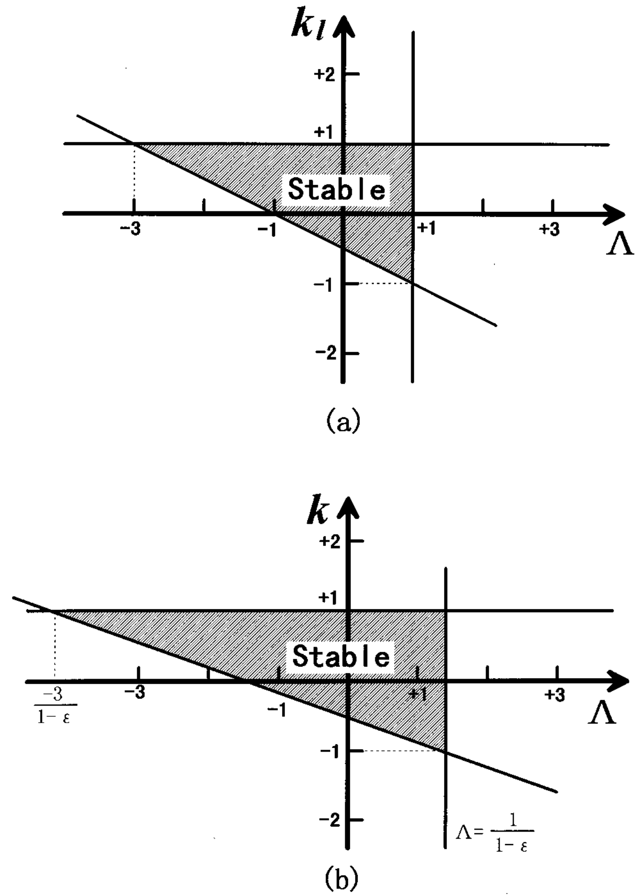

FIG. 1. Stable regions on the Λ-k plane. (a) The stable region of Eq. (9). (b) The stable region of Eq. (13).

is the slope of the local map f at the unstable fixed point x_f. Matrix (6) has eigenvalues that are given as solutions of the equation

$$\lambda^2-\lambda(\Lambda+k_l)+k_l=0. \tag{9}$$

Jury's criterion [35] allows us to derive the stability condition of the first site [see Fig. 1(a)]:

$$\begin{aligned} -3&<\Lambda<1,\\ 0&<1+\Lambda+2k_l,\\ k_l&<1. \end{aligned} \tag{10}$$

Conditions (10) are equal to the results of the papers [4,14].

Next, we consider the stability of the ith site $x_n(i)$. If the site $x_n(i-1)$ is stabilized onto x_f, the equation of the ith site and the ith local controller will be described by

$$x_{n+1}(i)=(1-\varepsilon)f[x_n(i)]+\varepsilon x_f+k[x_n(i)-x_{n-1}(i)].$$

This system can be linearized as

$$\begin{bmatrix} y_{n+1}(i)\\ z_{n+1}(i)\end{bmatrix}=\begin{bmatrix} (1-\varepsilon)\Lambda+k & -k\\ 1 & 0\end{bmatrix}\begin{bmatrix} y_n(i)\\ z_n(i)\end{bmatrix}, \tag{11}$$

$$y_n(i)=x_n(i)-x_f, \quad z_n(i)=x_{n-1}(i)-x_f. \tag{12}$$

The stability condition of the characteristic equation of matrix (11),

$$\lambda^2 - \lambda\{(1-\varepsilon)\Lambda + k\} + k = 0, \tag{13}$$

is given by the inequalities

$$-\frac{3}{1-\varepsilon} < \Lambda < \frac{1}{1-\varepsilon},$$
$$0 < 2k + (1-\varepsilon)\Lambda + 1, \tag{14}$$
$$k < 1.$$

Conditions (14) are illustrated in Fig. 1(b).

The OCML has a propagation of activity in only one direction; therefore, the complexity of the dynamics can be reduced. The above simple stability conditions (10) and (14) are due to the reduction in the complexity. On the other hand, if the connection of the lattices was two way, the dynamics would be complicated. It should be noted that our simple results cannot be applied to two-way connection systems.

From the above analysis, we see that all sites converge on x_f in the order of the site number, if the following conditions are satisfied: (i) The first site satisfies conditions (10). (ii) The first site orbit $x_n(1)$ without control sometime visits the neighborhood of x_f. (iii) The other sites satisfy conditions (14). (iv) The other site orbits $x_n(i)$ without control sometime visit the neighborhood of x_f when the previous sites $x_n(i-1)$ have been stabilized on x_f. From Figs. 1(a) and 1(b), we note that there exist local controllers (5) such that the conditions (10) and (14) are satisfied if and only if the slope Λ exists within the following range:

$$-3 < \Lambda < 1.$$

This clarifies that local controllers (5) cannot stabilize a class of the OCML (i.e., $\Lambda > 1$ or $\Lambda < -3$). Furthermore, the stable regions shown in Figs. 1(a) and 1(b) allow us to design the local controllers theoretically. Conditions (10) and (14) are satisfied if the feedback gains of the local controllers are chosen from

$$-\frac{\Lambda}{2} - \frac{1}{2} < k_l < 1, \quad -\frac{1-\varepsilon}{2}\Lambda - \frac{1}{2} < k < 1. \tag{15}$$

This is a procedure to design the local controllers by *a priori* knowledge of Λ and ε. If the slope Λ and the coupling strength ε were obtained in advance, inequalities (15) would provide a useful design procedure. In most practical situations, however, it is difficult to obtain a precise Λ and ε. For such situations, we have to consider the robust control problem to design the controllers by the uncertain information.

Let us assume that the uncertain information

$$\varepsilon_{\min} < \varepsilon < \varepsilon_{\max}, \quad \Lambda_{\min} < \Lambda < \Lambda_{\max} \tag{16}$$

is obtained in advance. The problem to be investigated is to design the local controllers such that the control systems (6) and (11), including uncertainties (16), are stable. To begin with, we derive a sufficient condition for the existence of the robust local controllers. We notice that if the lower and upper limits of Λ satisfy

$$-3 < \Lambda_{\min}, \quad \Lambda_{\max} < 1, \tag{17}$$

then there exist local controllers (5) such that conditions (10) and (14) are satisfied. If conditions (17) are satisfied, then we can design robust local controllers. The feedback gains of local controllers (5) should be chosen from

$$-\frac{1}{2}\Lambda_{\min} - \frac{1}{2} < k_l < 1, \quad -\frac{1-\varepsilon_{\min}}{2}\Lambda_{\min} - \frac{1}{2} < k < 1. \tag{18}$$

Let us summarize the procedure to design the robust local controllers as follows: (i) We obtain the uncertain information [Eq. (16)]. (ii) If the uncertain information satisfies conditions (17), we go to the next step; otherwise it is impossible to design the robust local controllers. (iii) We determine the feedback gains k_l and k by Eqs. (18).

IV. CONTROLLING A ONE-WAY COUPLED LOGISTIC LATTICE

The above theoretical results depend only on the slope Λ and the coupling strength ε; hence various one-dimensional chaotic maps can be used as the local map f in the DDFC system. This paper uses the logistic map

$$f[x] = px(1-x)$$

as a local map, since the map has already been investigated in detail.

The one-way coupled logistic map is described as

$$x_{n+1}(1) = px_n(1)(1-x_n(1)),$$
$$x_{n+1}(i) = (1-\varepsilon)px_n(i)(1-x_n(i)) + \varepsilon px_n(i-1)$$
$$\times(1-x_n(i-1)).$$

It is obvious that the slope Λ at the UFP is given by

$$\Lambda = 2 - p.$$

From a practical viewpoint, we shall consider the following assumptions: the local map f is unknown; the desired unstable homogenous state is unknown; the uncertain information [Eqs. (16)] is obtained in advance.

Let us consider the one-way coupled logistic lattice with $N=60$, $p=3.91$, and $\varepsilon=0.1$. We do not know this information (N,p,ε) when we design the controllers; on the other hand, the uncertain information $\Lambda_{\min}=-1.95$, $\Lambda_{\max}=-1.50$, $\varepsilon_{\min}=0.05$, and $\varepsilon_{\max}=0.20$ is obtained in advance. Since $-3<\Lambda_{\min}$ and $\Lambda_{\max}<1$, there exist robust local controllers such that the uncertain control system is stable. The feedback gains can be chosen from $0.475<k_l<1$ and $0.426\,25<k<1$; then we set $k_l=k=0.5$. It is undesirable for the control signal to be large when the local orbit is far from the UFP, since such a signal may make the control system fall into a divergence regime. To this end, we employ local watchers for every site. Each local watcher is described as

$$u_n(i) = \begin{cases} u_n(i) & \text{if } |u_n(i)| < \nu \\ 0 & \text{if } |u_n(i)| > \nu, \end{cases}$$

where the threshold ν is a small positive value. Figure 2 shows the numerical control results. Figure 2(a) is the space-

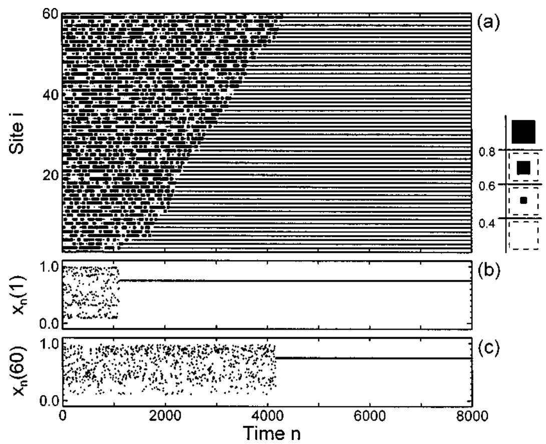

FIG. 2. Numerical control of the one-way coupled logistic lattice with $N=60$, $p=3.91$, and $\varepsilon=0.1$. The local controllers and watchers are set as $k_l=k=0.5$ and $\nu=0.02$. The controllers and watchers start to work at $n=1000$. (a) The space-time plot. (b) The local time series of the first site $x_n(1)$. (c) The local time series of the 60th site $x_n(60)$.

time plot for the 60 coupled one-way logistic maps with local controllers and watchers. The system runs freely [i.e., $u_n(i)=0$] until $n=1000$, and then the controllers and watchers start to work at $n=1000$. Initially the first site $x_n(1)$ converges on x_f by the first local controller. After that, all the sites $x_n(i)$ $(i=2,\ldots,60)$ converge on x_f in the order of the site number. Figures 2(b) and 2(c) show the local time series on the edge of the OCML [i.e., $x_n(1)$, $x_n(60)$].

The feedback pining technique [26–28] employed distributed controllers at several spatial locations; conversely, the DDFC proposed in the present paper requires a local controller for every site. From our theoretical results, we know that if at least one local controller, that is the Mth controller, is cut off [i.e., $u_n(M)=0$], then the higher sites $x_n(i)$ $(i=M,M+1,\ldots,N)$ cannot be stabilized. Figure 3(a) shows the space-time plot for the same system as Fig. 2. We cut off

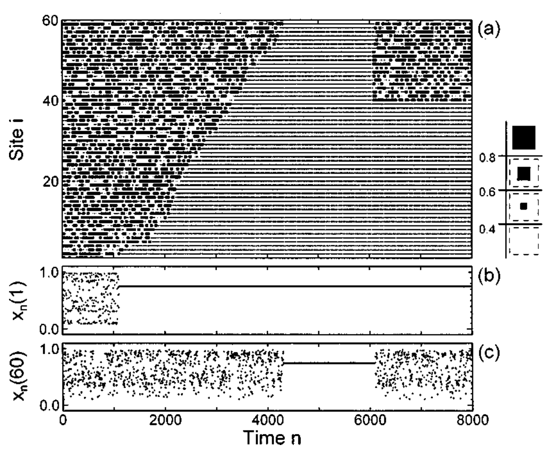

FIG. 3. Numerical control of the one-way coupled logistic lattice with $N=60$, $p=3.91$, and $\varepsilon=0.1$. The local controllers and watchers are set as $k_l=k=0.5$ and $\nu=0.02$. The controllers and watchers start to work at $n=1000$, and we cut off the 40th local controller [i.e., $u_n(40)=0$] at $n=8000$. (a) The space-time plot. (b) The local time series of the first site $x_n(1)$. (c) The local time series of the 60th site $x_n(60)$.

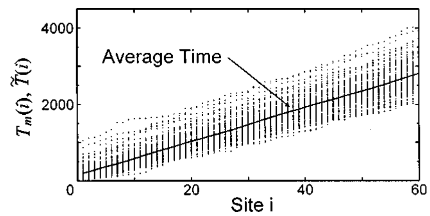

FIG. 4. Average time to achieve the stabilization of the same system as Fig. 2. The dots show the time $T_m(i)$ to achieve the stabilization of the ith site for the mth initial condition. The bold line represents the average time $\tilde{T}(i)$ for $T_m(i)$.

the local controller of the 40th site at $n=8000$, while other controllers and watchers continue to work. After $n=8000$, the 40th site and higher sites $x_n(i)$ $(i=41,42,\ldots,60)$ behave chaotically. We can see that higher controllers cannot stabilize the higher sites due to chaotic behavior of the 40th site. Figures 3(b) and 3(c) are the local time series of $x_n(1)$ and $x_n(60)$.

As can be seen from Fig. 2, all sites $x_n(i)$ converge on x_f in the order of the site number. Now we focus on the time to achieve the stabilization of the homogenous state. We estimate the average time for each local site in Fig. 4, starting from 100 random initial conditions on the chaotic attractor. The dots in Fig. 4 indicate the time $T_m(i)$ that is required to achieve the stabilization of $x_n(i)$ for the mth initial condition. The bold line represents the average time $\tilde{T}(i)$ to achieve the stabilization:

$$\tilde{T}(i)=\frac{1}{100}\sum_{m=1}^{100} T_m(i).$$

As is evident, the average time $\tilde{T}(i)$ must be proportional to the site number. From the above numerical results, the average time $\tilde{T}(i)$ can be estimated as

$$\tilde{T}(i)\simeq 42.1i+198.2.$$

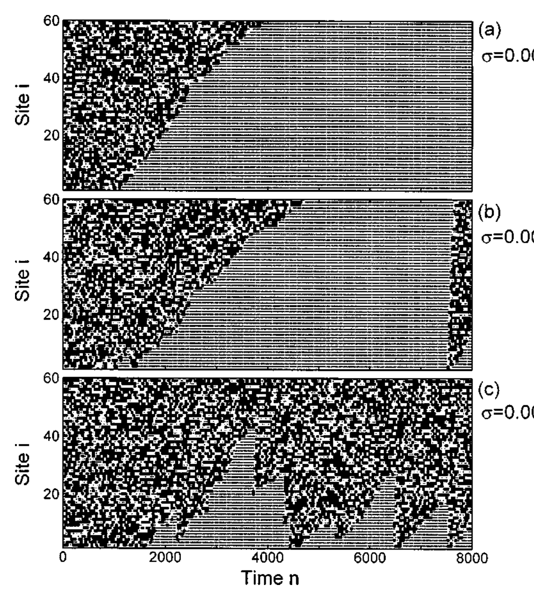

FIG. 5. Numerical control of the same system as Fig. 2 with the noisy signal $\sigma\eta_n$. (a) $\sigma=0.005$. (b) $\sigma=0.006$. (c) $\sigma=0.007$.

Although the constant values of this experimental equation depend on the local map *f*, the coupling strength ε, and the watcher's threshold ν, the linear relation may be retained for various values.

Stabilization in a noisy environment is an important issue in view of practical applications. We add the noisy signal $\sigma\eta_n$ to all the sites, where η_n is the random sequences in the range [0,1]. Figure 5 shows the space-time plots of the same system as Fig. 2 for three-level noise amplitudes ($\sigma = 0.005$, 0.006, and 0.007). It can be seen that if the amplitude σ is 0.005 or less, then the stabilization of all the sites is achieved successfully [see Fig. 5(a)]. It should be noted that the sites never converge on x_f, but always wander around it. On the other hand, if the amplitude σ is 0.006 or more, the stabilization cannot be achieved [see Figs. 5(b) and 5(c)]. The crisis of the noise amplitude depends on the local map *f*, the coupling strength ε, and the watcher's threshold ν.

V. CONCLUSIONS

In this paper we have shown that our DDFC technique can suppress turbulent behavior in the OCML. A theoretical analysis gave the necessary and sufficient conditions for the control system to be stable. The conditions allowed us to derive the procedure to design the robust local controllers. On the other hand, the conditions clarified that the DDFC technique cannot stabilize a class of the OCML (i.e., $\Lambda < -3$ or $\Lambda > 1$). This is an inherent weak point of the DFC technique [4,11–15]. A discussion of this issue would carry us too far from the purpose of this paper, thus we did not try to overcome this weak point. Furthermore, the theoretical results were verified by some numerical simulations.

The present paper focused on the stabilization of the unstable homogeneous state of the OCML. The control of complex patterns or high-period periodic orbits is also an instructive issue for studies of associate memory in dynamical systems. There is room for further investigation.

Our DDFC technique could be applied to a stabilization of networks of one-way coupled map lattices [36]. The stability of the networks could be analyzed by the results of the present paper. We will investigate this stability in detail, and report on this elsewhere.

ACKNOWLEDGMENTS

This work was partially supported by a Grant-in-Aid for Scientific Research from the Ministry of Education, Sports, Science and Culture, Government of Japan (Grant No. 09750076).

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